On One Property of Natural Number

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Abstract. *In the present work summarizes a remarkable property of natural numbers, seen and proven but not published by the Farm. We proved that for n = 2 and n = 3 we can find natural numbers sandwiched between n − 1 degree and n-th degree, and for n = 2 there are infinitely many such numbers, and for n = 3 there is a single natural number 26.*

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In 2014 we found in [1,68] one Fermat statement without proof of the remarkable property of the natural number 26 enclosed {clutched) between two numbers, one of which is a square ($25 = 5^2$) and the other is a cube ($27 = 3^3$). Fermat managed to prove that 26 was indeed the only number between a square and a cube. Fermat reported [1,68] about the unique property of number 26 to the mathematical community (usually without publishing the proof obtained), and call to prove it. The proof of the statement proved extremely difficult that the English mathematicians Wallis and Digby were forced to confess defeat.

We must pay proper to the honesty of Wallis and Digby, who did not follow the exemple of D.Cardano, who published under his own name the formula for solving the reduced cubic equation, communicated to him by N. Tartalia [1,50]. Simon Singh [1,163] following the example of A.Wiles, who allegedly proved [2] Fermat's Last Theorem, reduces the solution of this problem to the question of whether an elliptic curve $y^2 = x^3 + ax^2 + bx + c$, $a = 0$, $b = 0$, $c = -2$ has a unique solution $x = 3$, $y = 5$.

Just like Fermat, who generalized the Pythagorean equation in the form of Diophantine equations in the fields of the book "Arithmetic" and formulated his hypothesis about these equations: it is impossible to decompose neither a cube into two cubes, nor a biquadrate into two biquadrates and in General any degree greater than a square into two degrees with the same exponent, which has been proved [4-5] we decided to generalize the problem of the remarkable property of the number 26: find naturaj numbers sandwiched between n - 1 degree and n-th degree and proved that such numbers exist in the cases of $n = 2$ and $n = 3$.

Evidence, in case $n = 2$ for any $m > 1$ we have $m^2 - (m^2 - 2) = 2$. Thus, for $n = 2$ there is an infinite number of positive integers sandwiched between the square and the first degree.

For $n == 3$ this problem reduces to solution the algebraic equation

$$x^3 - (x + k)^2 = 2, \quad (1)$$

where $k > 0$ is a natural number. If $k = 1$, we obtain $x^3 - (x + 1)^2 = 2 \Leftrightarrow x^3 - x^2 - 2x - 3 = 0$. If the resulting equation has natural solution, then they should be among the natural divisors of 1,3 of the free term of the equation. Check make sure that there are no natural solutions,

If $k = 2$ we get $x^3 - (x + 2)^2 = 2 \Leftrightarrow x^3 - x^2 - 4x - 6 = 0$. If the resulting equation has natural solutions, they are among the natural divisors 1,2,3,6 of the free term of the equation. Check to make sure that the only number 3 is the solution of the equation. Thus, when $k = 2$, we get the number 26, sandwiched between 25 and 27.

If $k = 3$, we obtain $x^3 - (x + 3)^2 = 2 \Leftrightarrow x^3 - x^2 - 6x - 11 = 0$. If the resulting equation has natural solutions, then they are among the 1,11 divisor of the free term of the equation. Check make sure that there are no natural solutions. Suppose that if $k = 4,5,\ldots,m$ there are no natural solutions of the equation (1) and for $k = m + 1$ the divisor of the free term of the equation $x^3 - (x + m + 1)^2 = 2$ is its solution. Then, according to the descent axiom [6] the natural divisor d_m of the free term of the equation $x^3 - (x + m + 1)^2 = 2$ is its solution. Then, according to the descent axiom [6] the natural divisor d_m of the free term of the equation $x^3 - (x + m)^2 = 2$ is its solution, which contradicts the inductive assumption. The obtained contradiction confirms that for $n = 3$ the only number 26 is sandwiched between a square and a cube, which was to be proved.

CONCLUSIONS

In 2015 we introduced the concept of a binary mathematical statement A_n depending on the natural argument [6] and formulated the descent axiom for such statements, with the help of which we closed [4-7] many open problems in number theory, some of which are more than 2000 years sold. We believe that this state is natural, since it is known that non-Euclidean geometry has become generally recognized after more than 30 years since its discovery.

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