Algorithm of generation of prime numbers of form 4k+1 and 4k-1

Kochkarev B.S.

DOI: *10.5281/zenodo.3727646*

Leonard Euler tried to prove one of Fermat's most elegant remarks, the Prime number theorem [1, 73]. All Prime numbers except 2 are divided into numbers that are represented as 4k+1 and numbers that are represented as 4k-1, where k is some integer. Fermat's theorem on primes states that primes of the first group are always represantable as the sum of two squares, whereas primes of the second group are never representable as the sum of two squares. This property of primes is formulated elegantly and simply, but all attempts to prove that any Prime number $p \neq 2$ has this property encounter considerable difficulties. However, the Prime number 2 is also represented as the sum of two squares, for 2=1+1.

In 1749, after seven years of work on this, and almost 100 years after the death of Farm, Euler was able to prove this Prime number theorem [1, 73]. Since this statement is obviously binary [2], we also easily proved it using our descent axiom [3].

If we divide natural numbers into classes of residues modulo 4, and denote them according to [4] after $\bar{0}, \bar{1}, \bar{2}, \bar{3}$, all primes of the form 4k+1 will be in class of residue $\bar{1}$ and primes of the form 4k-1 in the class of residue $\bar{3}$. Obviously, if the natural number n is in the class of residue $\bar{i}$ modulo 4, then the number n+4k, where n is a natural number, is also in the residue class $\bar{i}$ modulo 4. This fact allows us to define an algorithm for constructing all primes of form 4k+1 and 4k-1. The smallest Prime number of the form 4k-1 is 3. The next Prime number of the form 4k-1 will be obtained by adding 4 to 3 as many times as possible until you get a Prime number and so on. If composite numbers are obtained in the process of applying the algorithm, they are excluded. For example, in our case, after the Prime number 3, the next Prime number will be 7, then 11, and after 11 we get 15, which is composite, so it is excluded. So, we get primes of the form 4k-1: 3, 7, 11, 19, 23, and so on. Similarly, primes of the form 4k+1 are generated. The first Prime number of the form 4k+1 is 5, then 13, then 17, 29, and so on.

References:

1. Singh S. Velikaya teorema Ferma. MTSHMO 2000, p. 288 (in Russian)
2. Kochkarev B.S. K metodu spuska Ferma. Problems of modern science and education. 2015, 11(41), s. 7-10 (in Russian)
3. Kochkarev B.S. Problema bliznetsov i drugie binarnye problemy. Problems of modern science and education. 2015, 11(41), s. 10-12 (in Russian)
4. Bukhstab A.A. Teoriya chisel. Pro0svetchenie. Moskva, 1966, c. 384 (in Russian)