UDC 517(076)

Linear Algebraic Systems with Complex Coefficients

Rakhshanda Dzhabarzadeh

We consider the algebraic system with n unknowns and with complex coefficients

$$\begin{cases} a_{0,i} = \sum_{k=1}^{n} a_{k,i} x_k \\ i = \overline{1,n} \end{cases} , \qquad (1)$$

where $a_{i,k} = b_{i,k} + ic'_{i,k}$ - the complex numbers, $x_1, x_2, \dots, x_n$ are unknowns. Our aim is to find the solutions of the system (1). In this article we represent the new way of finding of the solutions of the system (1).

To do this, we reduce the study of the system (1) to the study of an algebraic system with real coefficients constructed by the system (1).

We introduce the notations,

$$x_1 = \mu_1 + i\mu_2, .., x_k = \mu_{2(k-1)+1} + i\mu_{2k}, \ \dots, x_n = \mu_{2(n-1)+1} + i\mu_{2n} \qquad (2)$$

where $\mu_k (k = 1,2,\dots,2n)$ are real numbers.

Let $x_1, x_2, \dots, x_n$ is the solution of the system (1). The last means that (1) satisfies.

We use formula (2) and the equalities $a_{ik} = b_{i,k} + ic'_{i,k}$ in (1).

Each equation from (1) after separation into real and imaginary parts is two equations with 2n variables.

$$\begin{aligned} b_{0,i} &= b_{1,i}\mu_1 - c_{1,i}\mu_2 + b_{2,i}\mu_3 - c_{2,i}\mu_4 + \dots + b_{n,i}\mu_{2n-1,i} - c_{n,i}\mu_{2n} = 0 \\ c_{0,i} &= c_{1,i}\mu_1 + b_{1,i}\mu_2 + c_{2,i}\mu_3 + b_{2,i}\mu_4 + \dots + c_{n,i}\mu_{2n-1,i} + b_{n,i}\mu_{2n} = 0 \end{aligned} \qquad (3)$$

$$i = 1,2,\dots,n$$

because $a_{i,k}x_k = (b_{i,k} + ic'_{i,k\mu})(\mu_{2i-1} + i\mu_{2i}) = b_{i,k}\mu_{2u-1} - c_{i,k}\mu_{2i} \quad i = 1,2,\dots,n; k = 1,2,\dots,n)$

Thus we obtain an algebraic system (3) with real coefficients and with 2n unknowns.

The main determinant Δ_0 of this system has the form,

$$\Delta_0 = \begin{vmatrix} b_{1,1} & -c_{1,1} & b_{2,1} & -c_{2,1} & . & b_{n,1} & -c_{n,1} \\ c_{1,1} & b_{1,1} & c_{2,1} & b_{2,1} & \dots & c_{n,1} & b_{n,1} \\ b_{1,2} & -c_{1,2} & b_{2,,2} & -c_{2,2} & \dots & b_{n,2} & -c_{n,2} \\ c_{1,2} & b_{1,2} & c_{2,2} & b_{2,2} & \dots & c_{n,2} & b_{n,2} \\ . & . & . & . & \dots & . & . \\ b_{1,n} & -c_{1,n} & b_{2,n} & -c_{2,n} & ,,, & b_{n,n} & -c_{n,n} \\ c_{1,n} & b_{1,n} & c_{2,n} & b_{2,n} & \dots & c_{n,n} & b_{n,n} \end{vmatrix}$$

We have the following: if the determinant Δ_0 is not equal to zero the system (3) has the solutions. Determinants $\Delta_i \ (i = 1,2,\dots,2n)$ are obtained by means replacing i-th column by the column from the elements $(b_{0,1}, c_{0,1}, b_{0,2}, c_{0,2}, \dots, b_{0,n}, c_{0,n}$

We designate through $\delta_i \ (i = 1,2,..,2n)$ expressions $\frac{\Delta_i}{\Delta_0} \ (i = 1,2,..,2n)$ s, accordingly. We have

$$\mu_i = \delta_i = \frac{\Delta_i}{\Delta_0} \ (i = 1,2,..,2n)$$

We proved the following: if $x_1, x_2, \ldots, x_n$ are the solution of the system (1) then $\Delta_0 \neq 0$ and all $x_i = \dfrac{\Delta_{2i-1}}{\Delta_0} + i\dfrac{\Delta_{2i}}{\Delta_o} \ (i = 1,2,..,n)$

Further we prove that the condition $\Delta_0 \neq 0$ is sufficient for the existence of solutions of the algebraic system (1). We build the determinants by the rules given system (1). Suppose the determinant $\Delta_0 \neq 0$. Then we may define all the solutions of the system (3).

Tem (3). Besides of, we have

$$-b_{0,i} + \sum_{k=1}^{n} b_{k,i}\frac{\Delta_{2k-1}}{\Delta_0} - \sum_{k=1}^{n} c_{k,i}\frac{\Delta_{2k}}{\Delta_o} = 0$$

$$-c_{0,i} + \sum_{k=0}^{n} c_{k,i}\frac{\Delta_{2k-1}}{\Delta_0} + \sum_{k=1}^{n} b_{k,i}\frac{\Delta_{2k}}{\Delta_0} = 0 \qquad (5)$$

$$i = 1,2,\ldots,2n$$

(5) is the algebraic factor. Multiplying each (2k-1)th equation from (5) on i and adding \(2k) th eqution from (5), we obtain $x_1, x_2, \ldots, x_n$ is the solution of the (2k) th equation from (5), we obtain $x_i = \dfrac{\Delta_{2i-1}}{\Delta_0} + i\dfrac{\Delta_{2i}}{\Delta_o} \ (i = 1,2,..,n)$ and $a_{ik} = b_{i,k} + ic'_{i,k}$

Repeating these procedures with each pair of equations having the same second index of coefficients we obtain the fulfilling of all equations of (1) are satisfied.

Theorem 1. The condition $\Delta_0 \neq 0$ is the necessary and sufficient for the existence of solutions of (1)

The proved statement is a consequence of the results from [1] and [2]. Considered in [1] a multi-parameter system requires for research methods of the spectral theory of multi-parameter system. We investigate an algebraic system (1) by elementary ways.

We represent more general result from [1] and [2]. Let be

$$A_k(\lambda)x_k = \lambda_1 A_{1,k}.x_k + \lambda_2 A_{2,k}x_k + \ldots + \lambda_n A_{n,k}x_k \qquad (6)$$

$$k = 1,2,\ldots,n$$

$A_{i,k}$ are normal bounded operators acting in a separable Hilbert space H_k. For each fixed k the operators $A_{i,k}$ commute pairwise. For such a system, a separation theorem and a spectral theorem are proved.

Let be $H = H_1 \otimes H_2 \otimes \ldots \otimes H_n$ the tensor product of spaces $H._i.$ H^2 is the tensor product of two copies of space H.

Consider the operators defined by the equalities

$$\delta\tilde{x} = \sum_{i=0}^{2n} \alpha_i \delta_i \tilde{x} = \begin{pmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_{2k} & \alpha_{2k+1} & \cdots & \alpha_{2n} \\ \frac{A_{0,1}+A^*_{0,1}}{2}x_1 & \frac{A_{1,1}+A^*_{1,1}}{2}x_1 & \cdots & \frac{A^*_{k,1}-A_{k,1}}{2i}x_1 & \frac{A^*_{k+1,`1}-A_{k+1,1}}{2i}x_1 & \cdots & \frac{A^*_{n,1}-A_{n,,1}}{2i}x_1 \\ . & . & \cdots & . & . & \cdots & . \\ \frac{A_{0,n}+A^*_{0,n}}{2}x_n & \frac{A_{1,n}+A^*_{1,n}}{2}x_n & \cdots & \frac{A^*_{k,n}-A_{k,n}}{2i}x_n & \frac{A^*_{n,,k+1}+A_{k+1,n}}{2}x_n & \cdots & \frac{A^*_{n,n}-A_{n,n}}{2i}x_n \\ \frac{A_{0,1}-A^*_{0,1}}{2i}x_{n+1} & \frac{A_{1,1}-A^\circ_{1,1}}{2i}x_{n+1} & \cdots & \frac{A_{k,1}+A^*_{k,1}}{2}x_{n+1} & \frac{A_{k+1,1}-A^*_{k+1,1}}{2i}x_{n+1} & \cdots & \frac{A_{n,1}+A^*_{n,1}}{2i}x_{n+1} \\ . & . & \cdots & . & . & \cdots & . \\ \frac{A_{0,n}-A^*_{0,n}}{2i}x_{2n} & \frac{A_{1,n}-A^*_{1,n}}{2i}x_{2n} & \cdots & \frac{A_{k,n}+A^*_{k,n}}{2}x_{2n} & \frac{A_{k+1,n}-A^*_{k+1,n}}{2i}x_{2n} & \cdots & \frac{A_{n,n}+A^\bullet_{n,n}}{2}x_{2n} \end{pmatrix} \qquad (7)$$

on decomposable tensors $\tilde{x} = x_1 \otimes x_2 \otimes \ldots \otimes x_n \otimes x_1 \otimes \ldots \otimes x_n \in H^2$

Let $\alpha_i \ (i = 0,1,2,..,2n)$ be arbitrary complex numbers. For each fixed i operator δ_i is determined from (6), when $\alpha_0 = \alpha_1 = \ldots = \alpha_{i-1} = \alpha_{i+1} = \ldots = \alpha_{2n} = 0,\ \alpha_i = 1$. On all other elements of the space H operators δ_i are defined by linearity and continuity.

Theorem 2. Let be $\delta_0 \neq 0$, then for the eigencalue $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ of the system (6) we have $\gamma_k\tilde{x} = \left(\delta_0^{-1}\delta_{2k-1} + i\delta_0^{-1}\delta_{2k}\right)\tilde{x} = \lambda_k\tilde{x}$]

If $H = R$, $A_{i,k}$ are complex numbers $a_{i,k}$, $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ is unknowns we have system(1).

Reference:

1.Аллахвердиев Дж.Э., Джабарзаде Р.М. О спектре многопараметрической системы испектральная теорема. ДАН. СССР,1988,т.300, 1, pp.101, 2015

2.Dzhabar-zadeh R.M. Spectral theory of two parameter system in finite dimensional space, Transactions of AS Azerbaijan, V.XVIII, №3-4, 1998, p. 12-18.

3.Dzhabar – zadeh R.M. Spectral theory of multiparameter system of opera-

tors in Hilbert space, Transactions of Sciences of Azerbaijan, V.XIX, №1-2, 1999, pp.33-40, Baku “Elm” Publishing House.

4.Рахшанда Джабарзаде Нелинейные алгебраические системы уравнений, изд. Lambert Academic Publishing.